9.4 Special Right Triangles

What *you should learn*

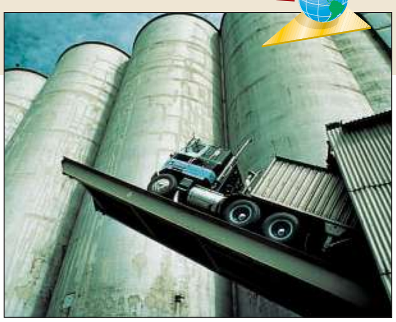
GOAL 1 Find the side lengths of special right triangles.

GOAL 2 Use special right triangles to solve **real-life** problems, such as finding the side lengths of the triangles in the spiral quilt design in **Exs. 31–34**.

Why *you should learn it*

▼ To use special right triangles to solve **real-life** problems, such as finding the height of a tipping platform in **Example 4**.

REAL LIFE



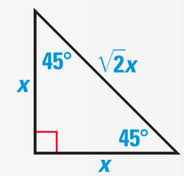
GOAL 1 SIDE LENGTHS OF SPECIAL RIGHT TRIANGLES

Right triangles whose angle measures are 45°-45°-90° or 30°-60°-90° are called **special right triangles**. In the Activity on page 550, you may have noticed certain relationships among the side lengths of each of these special right triangles. The theorems below describe these relationships. Exercises 35 and 36 ask you to prove the theorems.

THEOREMS ABOUT SPECIAL RIGHT TRIANGLES

THEOREM 9.8 ***45°-45°-90° Triangle Theorem***

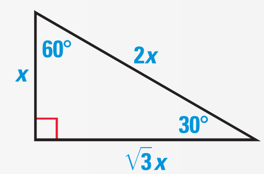
In a 45°-45°-90° triangle, the hypotenuse is $\sqrt{2}$ times as long as each leg.



Hypotenuse $= \sqrt{2} \cdot$ leg

THEOREM 9.9 ***30°-60°-90° Triangle Theorem***

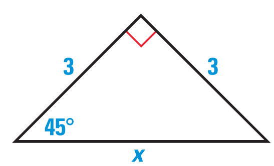
In a 30°-60°-90° triangle, the hypotenuse is twice as long as the shorter leg, and the longer leg is $\sqrt{3}$ times as long as the shorter leg.



Hypotenuse $= 2 \cdot$ shorter leg
Longer leg $= \sqrt{3} \cdot$ shorter leg

EXAMPLE 1 *Finding the Hypotenuse in a 45°-45°-90° Triangle*

Find the value of x.



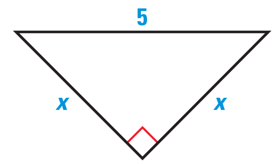
SOLUTION

By the Triangle Sum Theorem, the measure of the third angle is 45°. The triangle is a 45°-45°-90° right triangle, so the length x of the hypotenuse is $\sqrt{2}$ times the length of a leg.

Hypotenuse $= \sqrt{2} \cdot$ leg	45°-45°-90° Triangle Theorem
$x = \sqrt{2} \cdot 3$	Substitute.
$x = 3\sqrt{2}$	Simplify.

EXAMPLE 2 *Finding a Leg in a 45°-45°-90° Triangle*

Find the value of x.



SOLUTION

Because the triangle is an isosceles right triangle, its base angles are congruent. The triangle is a 45°-45°-90° right triangle, so the length of the hypotenuse is $\sqrt{2}$ times the length x of a leg.

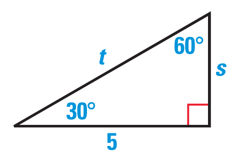
$\text{Hypotenuse} = \sqrt{2} \cdot \text{leg}$	45°-45°-90° Triangle Theorem
$5 = \sqrt{2} \cdot x$	Substitute.
$\frac{5}{\sqrt{2}} = \frac{\sqrt{2}x}{\sqrt{2}}$	Divide each side by $\sqrt{2}$.
$\frac{5}{\sqrt{2}} = x$	Simplify.
$\frac{\sqrt{2}}{\sqrt{2}} \cdot \frac{5}{\sqrt{2}} = x$	Multiply numerator and denominator by $\sqrt{2}$.
$\frac{5\sqrt{2}}{2} = x$	Simplify.

EXAMPLE 3 *Side Lengths in a 30°-60°-90° Triangle*

STUDENT HELP

INTERNET
HOMEWORK HELP
Visit our Web site www.mcdougallittell.com for extra examples.

Find the values of s and t.



SOLUTION

Because the triangle is a 30°-60°-90° triangle, the longer leg is $\sqrt{3}$ times the length s of the shorter leg.

$\text{Longer leg} = \sqrt{3} \cdot \text{shorter leg}$	30°-60°-90° Triangle Theorem
$5 = \sqrt{3} \cdot s$	Substitute.
$\frac{5}{\sqrt{3}} = \frac{\sqrt{3} \cdot s}{\sqrt{3}}$	Divide each side by $\sqrt{3}$.
$\frac{5}{\sqrt{3}} = s$	Simplify.
$\frac{\sqrt{3}}{\sqrt{3}} \cdot \frac{5}{\sqrt{3}} = s$	Multiply numerator and denominator by $\sqrt{3}$.
$\frac{5\sqrt{3}}{3} = s$	Simplify.

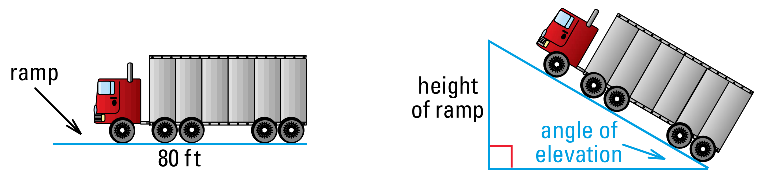
The length t of the hypotenuse is twice the length s of the shorter leg.

$\text{Hypotenuse} = 2 \cdot \text{shorter leg}$	30°-60°-90° Triangle Theorem
$t = 2 \cdot \frac{5\sqrt{3}}{3}$	Substitute.
$t = \frac{10\sqrt{3}}{3}$	Simplify.

GOAL 2 USING SPECIAL RIGHT TRIANGLES IN REAL LIFE

EXAMPLE 4 *Finding the Height of a Ramp*

TIPPING PLATFORM A tipping platform is a ramp used to unload trucks, as shown on page 551. How high is the end of an 80 foot ramp when it is tipped by a 30° angle? by a 45° angle?



SOLUTION

When the angle of elevation is 30°, the height h of the ramp is the length of the shorter leg of a 30°-60°-90° triangle. The length of the hypotenuse is 80 feet.

$80 = 2h$ 30°-60°-90° Triangle Theorem

$40 = h$ Divide each side by 2.

When the angle of elevation is 45°, the height of the ramp is the length of a leg of a 45°-45°-90° triangle. The length of the hypotenuse is 80 feet.

$80 = \sqrt{2} \cdot h$ 45°-45°-90° Triangle Theorem

$\frac{80}{\sqrt{2}} = h$ Divide each side by $\sqrt{2}$.

$56.6 \approx h$ Use a calculator to approximate.

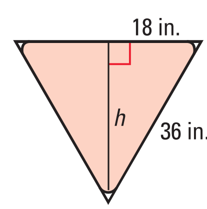
▶ When the angle of elevation is 30°, the ramp height is 40 feet. When the angle of elevation is 45°, the ramp height is about 56 feet 7 inches.

EXAMPLE 5 *Finding the Area of a Sign*

ROAD SIGN The road sign is shaped like an equilateral triangle. Estimate the area of the sign by finding the area of the equilateral triangle.

SOLUTION

First find the height h of the triangle by dividing it into two 30°-60°-90° triangles. The length of the longer leg of one of these triangles is h. The length of the shorter leg is 18 inches.



$h = \sqrt{3} \cdot 18 = 18\sqrt{3}$ 30°-60°-90° Triangle Theorem

Use $h = 18\sqrt{3}$ to find the area of the equilateral triangle.

$$\text{Area} = \frac{1}{2}bh = \frac{1}{2}(36)\left(18\sqrt{3}\right) \approx 561.18$$

▶ The area of the sign is about 561 square inches.

GUIDED PRACTICE

Vocabulary Check ✓

1. What is meant by the term *special right triangles*?

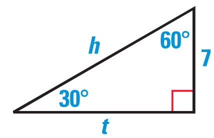
Concept Check ✓

2. **CRITICAL THINKING** Explain why any two 30°-60°-90° triangles are similar.

Use the diagram to tell whether the equation is *true* or *false*.

3. $t = 7\sqrt{3}$ **4.** $t = \sqrt{3h}$ **5.** $h = 2t$

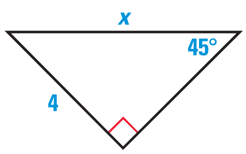
6. $h = 14$ **7.** $7 = \frac{h}{2}$ **8.** $7 = \frac{t}{\sqrt{3}}$



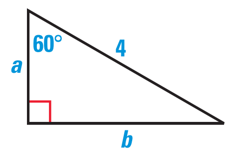
Skill Check ✓

Find the value of each variable. Write answers in simplest radical form.

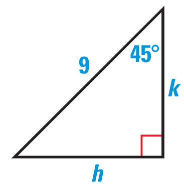
9.



10.



11.



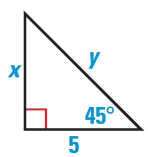
PRACTICE AND APPLICATIONS

STUDENT HELP

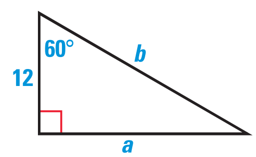
Extra Practice to help you master skills is on p. 820.

USING ALGEBRA **Find the value of each variable. Write answers in simplest radical form.**

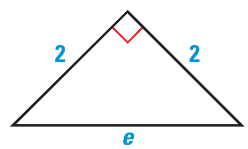
12.



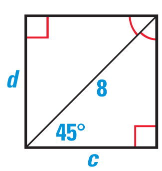
13.



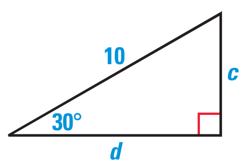
14.



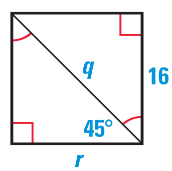
15.



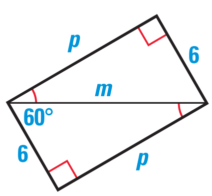
16.



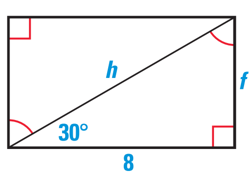
17.



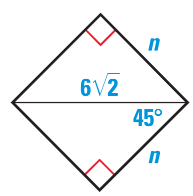
18.



19.



20.



STUDENT HELP

HOMEWORK HELP

Example 1: Exs. 12–23
Example 2: Exs. 12–23
Example 3: Exs. 12–23
Example 4: Exs. 28–29, 34
Example 5: Exs. 24–27

FINDING LENGTHS **Sketch the figure that is described. Find the requested length. Round decimals to the nearest tenth.**

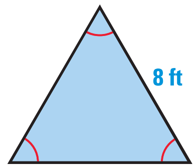
21. The side length of an equilateral triangle is 5 centimeters. Find the length of an altitude of the triangle.

22. The perimeter of a square is 36 inches. Find the length of a diagonal.

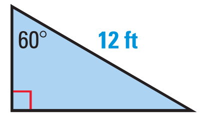
23. The diagonal of a square is 26 inches. Find the length of a side.

FINDING AREA **Find the area of the figure. Round decimal answers to the nearest tenth.**

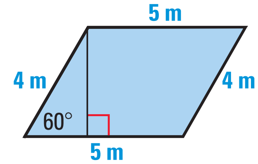
24.



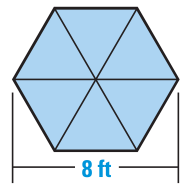
25.



26.



27. **AREA OF A WINDOW** A hexagonal window consists of six congruent panes of glass. Each pane is an equilateral triangle. Find the area of the entire window.

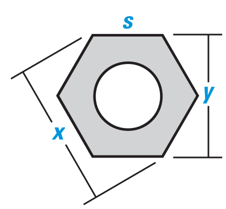


JEWELRY **Estimate the length *x* of each earring.**

28.

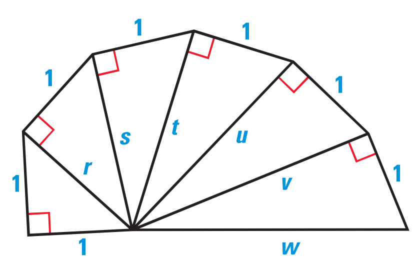
29.

30. **TOOLS** Find the values of x and y for the hexagonal nut shown at the right when $s = 2$ centimeters. (*Hint:* In Exercise 27 above, you saw that a regular hexagon can be divided into six equilateral triangles.)



LOGICAL REASONING **The quilt design in the photo is based on the pattern in the diagram below. Use the diagram in Exercises 31–34.**

Wheel of Theodorus



31. Find the values of r, s, t, u, v, and w. Explain the procedure you used to find the values.

32. Which of the triangles, if any, is a 45°-45°-90° triangle?

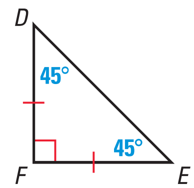
33. Which of the triangles, if any, is a 30°-60°-90° triangle?

34. **USING ALGEBRA** Suppose there are n triangles in the spiral. Write an expression for the hypotenuse of the nth triangle.

35. **PARAGRAPH PROOF** Write a paragraph proof of Theorem 9.8 on page 551.

GIVEN ▶ $\triangle DEF$ is a 45°-45°-90° triangle.

PROVE ▶ The hypotenuse is $\sqrt{2}$ times as long as each leg.

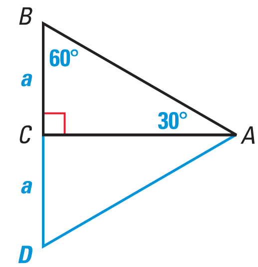


36. **PARAGRAPH PROOF** Write a paragraph proof of Theorem 9.9 on page 551.

GIVEN ▶ $\triangle ABC$ is a 30°-60°-90° triangle.

PROVE ▶ The hypotenuse is twice as long as the shorter leg and the longer leg is $\sqrt{3}$ times as long as the shorter leg.

Plan for Proof Construct $\triangle ADC$ congruent to $\triangle ABC$. Then prove that $\triangle ABD$ is equilateral. Express the lengths AB and AC in terms of a.



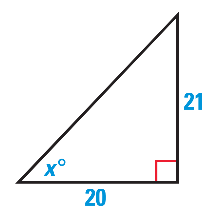
Test Preparation

37. **MULTIPLE CHOICE** Which of the statements below is true about the diagram at the right?

Ⓐ $x < 45$ Ⓑ $x = 45$

Ⓒ $x > 45$ Ⓓ $x \leq 45$

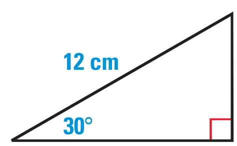
Ⓔ Not enough information is given to determine the value of x.



38. **MULTIPLE CHOICE** Find the perimeter of the triangle shown at the right to the nearest tenth of a centimeter.

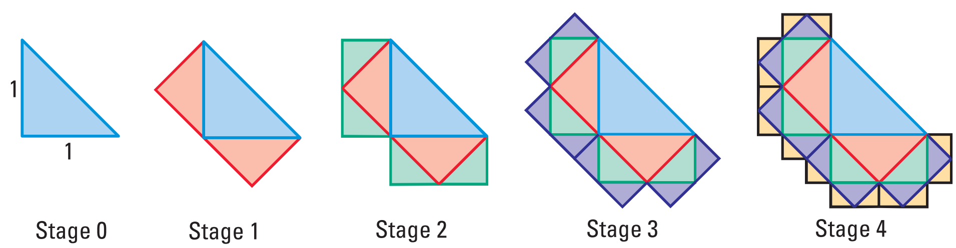
Ⓐ 28.4 cm Ⓑ 30 cm

Ⓒ 31.2 cm Ⓓ 41.6 cm



★ Challenge

VISUAL THINKING In Exercises 39–41, use the diagram below. Each triangle in the diagram is a 45°-45°-90° triangle. At Stage 0, the legs of the triangle are each 1 unit long.



39. Find the exact lengths of the legs of the triangles that are added at each stage. Leave radicals in the denominators of fractions.

40. Describe the pattern of the lengths in Exercise 39.

41. Find the length of a leg of a triangle added in Stage 8. Explain how you found your answer.

EXTRA CHALLENGE
www.mcdougallittell.com

MIXED REVIEW

42. FINDING A SIDE LENGTH A triangle has one side of 9 inches and another of 14 inches. Describe the possible lengths of the third side. (Review 5.5)

FINDING REFLECTIONS Find the coordinates of the reflection without using a coordinate plane. (Review 7.2)

43. $Q(-1, -2)$ reflected in the x-axis

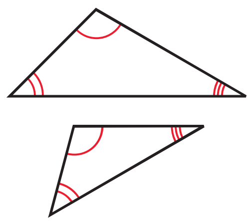
44. $P(8, 3)$ reflected in the y-axis

45. $A(4, -5)$ reflected in the y-axis

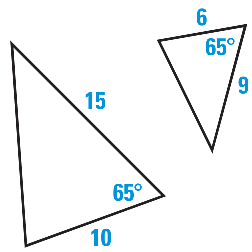
46. $B(0, 10)$ reflected in the x-axis

DEVELOPING PROOF Name a postulate or theorem that can be used to prove that the two triangles are similar. (Review 8.5 for 9.5)

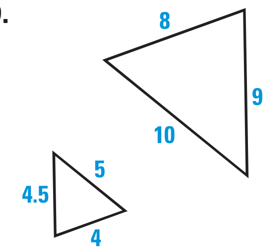
47.



48.



49.



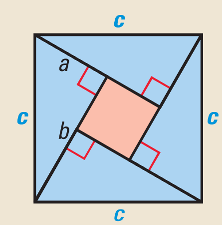
MATH & History — Pythagorean Theorem Proofs

APPLICATION LINK
www.mcdougallittell.com

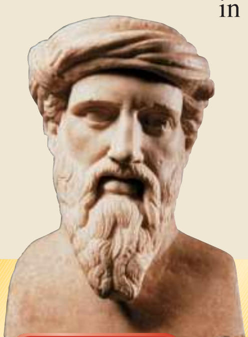
THEN **AROUND THE SIXTH CENTURY B.C.,** the Greek mathematician Pythagoras founded a school for the study of philosophy, mathematics, and science. Many people believe that an early proof of the Pythagorean Theorem came from this school.

NOW **TODAY,** the Pythagorean theorem is one of the most famous theorems in geometry. More than 100 different proofs now exist.

The diagram is based on one drawn by the Hindu mathematician Bhāskara (1114–1185). The four blue right triangles are congruent.



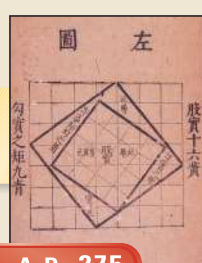
1. Write an expression in terms of a and b for the combined areas of the blue triangles. Then write an expression in terms of a and b for the area of the small red square.
2. Use the diagram to show that $a^2 + b^2 = c^2$. (*Hint:* This proof of the Pythagorean Theorem is similar to the one in Exercise 37 on page 540.)



c. 529 B.C.

School of Pythagoras is founded.

Chinese manuscript includes a diagram that can be used to prove the theorem.



c. A.D. 275

1876

Future U.S. President Garfield discovers a proof of the theorem.

Nicaraguan stamp commemorates the Pythagorean Theorem.

1971

9.5 Trigonometric Ratios

What you should learn

GOAL 1 Find the sine, the cosine, and the tangent of an acute angle.

GOAL 2 Use trigonometric ratios to solve **real-life** problems, such as estimating the height of a tree in **Example 6**.

Why you should learn it

▼ To solve **real-life** problems, such as in finding the height of a water slide in **Ex. 37**.

REAL LIFE

GOAL 1 FINDING TRIGONOMETRIC RATIOS

A **trigonometric ratio** is a ratio of the lengths of two sides of a right triangle. The word *trigonometry* is derived from the ancient Greek language and means measurement of triangles. The three basic trigonometric ratios are **sine, cosine,** and **tangent**, which are abbreviated as *sin*, *cos*, and *tan*, respectively.

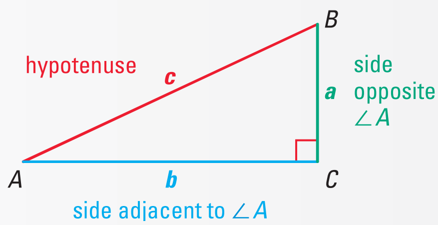
TRIGONOMETRIC RATIOS

Let $\triangle ABC$ be a right triangle. The sine, the cosine, and the tangent of the acute angle $\angle A$ are defined as follows.

$$\sin A = \frac{\text{side opposite } \angle A}{\text{hypotenuse}} = \frac{a}{c}$$

$$\cos A = \frac{\text{side adjacent to } \angle A}{\text{hypotenuse}} = \frac{b}{c}$$

$$\tan A = \frac{\text{side opposite } \angle A}{\text{side adjacent to } \angle A} = \frac{a}{b}$$



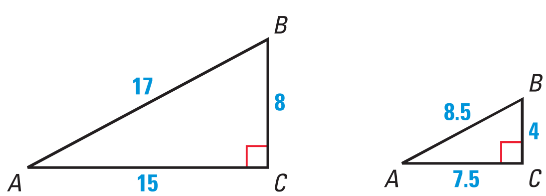
The value of a trigonometric ratio depends only on the measure of the acute angle, not on the particular right triangle that is used to compute the value.

EXAMPLE 1 *Finding Trigonometric Ratios*

Compare the sine, the cosine, and the tangent ratios for $\angle A$ in each triangle below.

SOLUTION

By the SSS Similarity Theorem, the triangles are similar. Their corresponding sides are in proportion, which implies that the trigonometric ratios for $\angle A$ in each triangle are the same.



	Large triangle	Small triangle
$\sin A = \dfrac{\text{opposite}}{\text{hypotenuse}}$	$\dfrac{8}{17} \approx 0.4706$	$\dfrac{4}{8.5} \approx 0.4706$
$\cos A = \dfrac{\text{adjacent}}{\text{hypotenuse}}$	$\dfrac{15}{17} \approx 0.8824$	$\dfrac{7.5}{8.5} \approx 0.8824$
$\tan A = \dfrac{\text{opposite}}{\text{adjacent}}$	$\dfrac{8}{15} \approx 0.5333$	$\dfrac{4}{7.5} \approx 0.5333$

Trigonometric ratios are frequently expressed as decimal approximations.

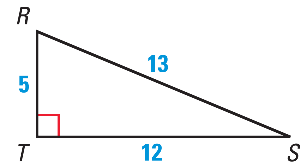
EXAMPLE 2 *Finding Trigonometric Ratios*

STUDENT HELP

INTERNET HOMEWORK HELP Visit our Web site www.mcdougallittell.com for extra examples.

Find the sine, the cosine, and the tangent of the indicated angle.

a. $\angle S$ **b.** $\angle R$



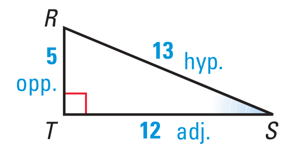
SOLUTION

a. The length of the hypotenuse is 13. For $\angle S$, the length of the opposite side is 5, and the length of the adjacent side is 12.

$$\sin S = \frac{\text{opp.}}{\text{hyp.}} = \frac{5}{13} \approx 0.3846$$

$$\cos S = \frac{\text{adj.}}{\text{hyp.}} = \frac{12}{13} \approx 0.9231$$

$$\tan S = \frac{\text{opp.}}{\text{adj.}} = \frac{5}{12} \approx 0.4167$$

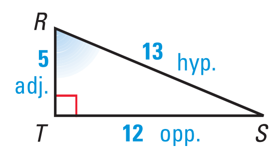


b. The length of the hypotenuse is 13. For $\angle R$, the length of the opposite side is 12, and the length of the adjacent side is 5.

$$\sin R = \frac{\text{opp.}}{\text{hyp.}} = \frac{12}{13} \approx 0.9231$$

$$\cos R = \frac{\text{adj.}}{\text{hyp.}} = \frac{5}{13} \approx 0.3846$$

$$\tan R = \frac{\text{opp.}}{\text{adj.}} = \frac{12}{5} = 2.4$$



You can find trigonometric ratios for 30°, 45°, and 60° by applying what you know about special right triangles.

EXAMPLE 3 *Trigonometric Ratios for 45°*

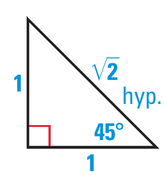
Find the sine, the cosine, and the tangent of 45°.

SOLUTION

STUDENT HELP

Study Tip
The expression sin 45° means the sine of an angle whose measure is 45°.

Begin by sketching a 45°-45°-90° triangle. Because all such triangles are similar, you can make calculations simple by choosing 1 as the length of each leg. From Theorem 9.8 on page 551, it follows that the length of the hypotenuse is $\sqrt{2}$.



$$\sin 45° = \frac{\text{opp.}}{\text{hyp.}} = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2} \approx 0.7071$$

$$\cos 45° = \frac{\text{adj.}}{\text{hyp.}} = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2} \approx 0.7071$$

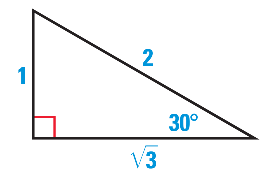
$$\tan 45° = \frac{\text{opp.}}{\text{adj.}} = \frac{1}{1} = 1$$

EXAMPLE 4 *Trigonometric Ratios for 30°*

Find the sine, the cosine, and the tangent of 30°.

SOLUTION

Begin by sketching a 30°-60°-90° triangle. To make the calculations simple, you can choose 1 as the length of the shorter leg. From Theorem 9.9 on page 551, it follows that the length of the longer leg is $\sqrt{3}$ and the length of the hypotenuse is 2.



$$\sin 30° = \frac{\text{opp.}}{\text{hyp.}} = \frac{1}{2} = 0.5$$

$$\cos 30° = \frac{\text{adj.}}{\text{hyp.}} = \frac{\sqrt{3}}{2} \approx 0.8660$$

$$\tan 30° = \frac{\text{opp.}}{\text{adj.}} = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3} \approx 0.5774$$

EXAMPLE 5 *Using a Calculator*

You can use a calculator to approximate the sine, the cosine, and the tangent of 74°. Make sure your calculator is in *degree mode.* The table shows some sample keystroke sequences accepted by most calculators.

Sample keystroke sequences	Sample calculator display	Rounded approximation
74 SIN or SIN 74 ENTER	0.961261695	0.9613
74 COS or COS 74 ENTER	0.275637355	0.2756
74 TAN or TAN 74 ENTER	3.487414444	3.4874

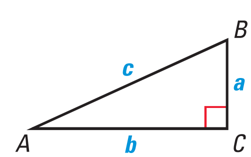
STUDENT HELP

Trig Table
For a table of trigonometric ratios, see p. 845.

If you look back at Examples 1–5, you will notice that the sine or the cosine of an acute angle is always less than 1. The reason is that these trigonometric ratios involve the ratio of a leg of a right triangle to the hypotenuse. The length of a leg of a right triangle is always less than the length of its hypotenuse, so the ratio of these lengths is always less than one.

Because the tangent of an acute angle involves the ratio of one leg to another leg, the tangent of an angle can be less than 1, equal to 1, or greater than 1.

TRIGONOMETRIC IDENTITIES A trigonometric identity is an equation involving trigonometric ratios that is true for all acute angles. You are asked to prove the following identities in Exercises 47 and 52:



$$(\sin A)^2 + (\cos A)^2 = 1$$

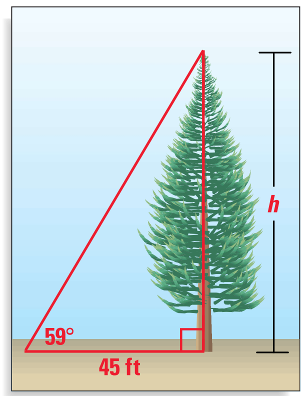
$$\tan A = \frac{\sin A}{\cos A}$$

GOAL 2 USING TRIGONOMETRIC RATIOS IN REAL LIFE

Suppose you stand and look up at a point in the distance, such as the top of the tree in Example 6. The angle that your line of sight makes with a line drawn horizontally is called the **angle of elevation**.

EXAMPLE 6 *Indirect Measurement*

FORESTRY You are measuring the height of a Sitka spruce tree in Alaska. You stand 45 feet from the base of the tree. You measure the angle of elevation from a point on the ground to the top of the tree to be 59°. To estimate the height of the tree, you can write a trigonometric ratio that involves the height h and the known length of 45 feet.



$$\tan 59^\circ = \frac{\text{opposite}}{\text{adjacent}}$$ **Write ratio.**

$$\tan 59^\circ = \frac{h}{45}$$ **Substitute.**

$$45 \tan 59^\circ = h$$ **Multiply each side by 45.**

$$45(1.6643) \approx h$$ **Use a calculator or table to find tan 59°.**

$$74.9 \approx h$$ **Simplify.**

▶ The tree is about 75 feet tall.

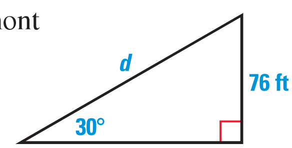
FOCUS ON CAREERS

FORESTRY Foresters manage and protect forests. Their work can involve measuring tree heights. Foresters can use an instrument called a *clinometer* to measure the angle of elevation from a point on the ground to the top of a tree.

INTERNET CAREER LINK
www.mcdougallittell.com

EXAMPLE 7 *Estimating a Distance*

ESCALATORS The escalator at the Wilshire/Vermont Metro Rail Station in Los Angeles rises 76 feet at a 30° angle. To find the distance d a person travels on the escalator stairs, you can write a trigonometric ratio that involves the hypotenuse and the known leg length of 76 feet.



$$\sin 30^\circ = \frac{\text{opposite}}{\text{hypotenuse}}$$ **Write ratio for sine of 30°.**

$$\sin 30^\circ = \frac{76}{d}$$ **Substitute.**

$$d \sin 30^\circ = 76$$ **Multiply each side by *d*.**

$$d = \frac{76}{\sin 30^\circ}$$ **Divide each side by sin 30°.**

$$d = \frac{76}{0.5}$$ **Substitute 0.5 for sin 30°.**

$$d = 152$$ **Simplify.**

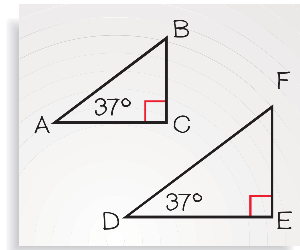
▶ A person travels 152 feet on the escalator stairs.

GUIDED PRACTICE

Vocabulary Check ✓

Concept Check ✓

In Exercises 1 and 2, use the diagram at the right.

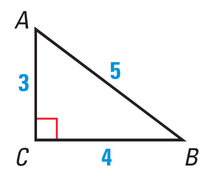


1. Use the diagram to explain what is meant by the *sine*, the *cosine*, and the *tangent* of $\angle A$.

2. ERROR ANALYSIS A student says that $\sin D > \sin A$ because the side lengths of $\triangle DEF$ are greater than the side lengths of $\triangle ABC$. Explain why the student is incorrect.

Skill Check ✓

In Exercises 3–8, use the diagram shown at the right to find the trigonometric ratio.

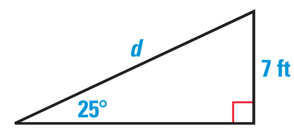


3. $\sin A$ **4.** $\cos A$

5. $\tan A$ **6.** $\sin B$

7. $\cos B$ **8.** $\tan B$

9. ESCALATORS One early escalator built in 1896 rose at an angle of 25°. As shown in the diagram at the right, the vertical lift was 7 feet. Estimate the distance d a person traveled on this escalator.



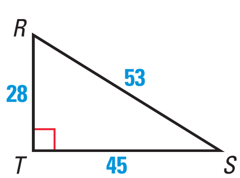
PRACTICE AND APPLICATIONS

STUDENT HELP

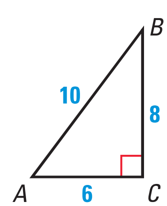
Extra Practice to help you master skills is on p. 820.

FINDING TRIGONOMETRIC RATIOS **Find the sine, the cosine, and the tangent of the acute angles of the triangle. Express each value as a decimal rounded to four places.**

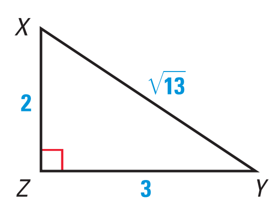
10.



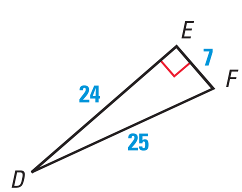
11.



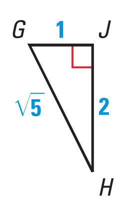
12.



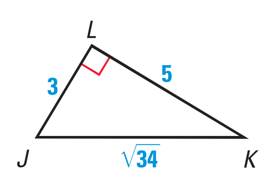
13.



14.



15.



STUDENT HELP

HOMEWORK HELP
Example 1: Exs. 10–15, 28–36
Example 2: Exs. 10–15, 28–36
Example 3: Exs. 34–36
Example 4: Exs. 34–36
Example 5: Exs. 16–27
Example 6: Exs. 37–42
Example 7: Exs. 37–42

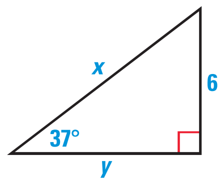
CALCULATOR **Use a calculator to approximate the given value to four decimal places.**

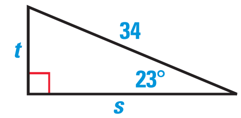
16. $\sin 48°$ **17.** $\cos 13°$ **18.** $\tan 81°$ **19.** $\sin 27°$

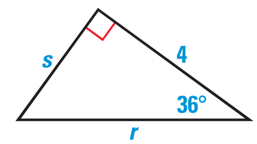
20. $\cos 70°$ **21.** $\tan 2°$ **22.** $\sin 78°$ **23.** $\cos 36°$

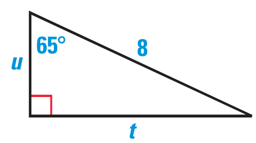
24. $\tan 23°$ **25.** $\cos 63°$ **26.** $\sin 56°$ **27.** $\tan 66°$

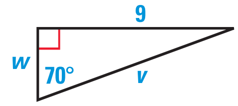
USING TRIGONOMETRIC RATIOS **Find the value of each variable. Round decimals to the nearest tenth.**

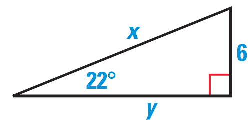
28.


29.


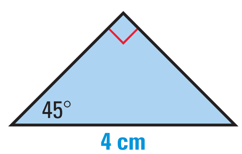
30.


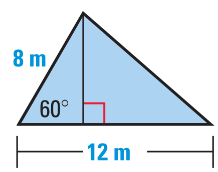
31.


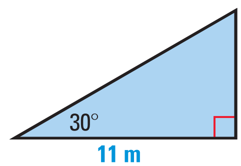
32.


33.


FINDING AREA **Find the area of the triangle. Round decimals to the nearest tenth.**

34.


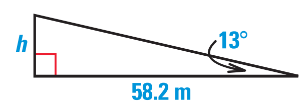
35.


36.


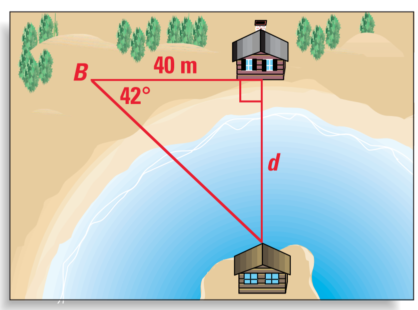
FOCUS ON APPLICATIONS

REAL LIFE **WATER SLIDE** Even though riders on a water slide may travel at only 20 miles per hour, the curves on the slide can make riders feel as though they are traveling much faster.

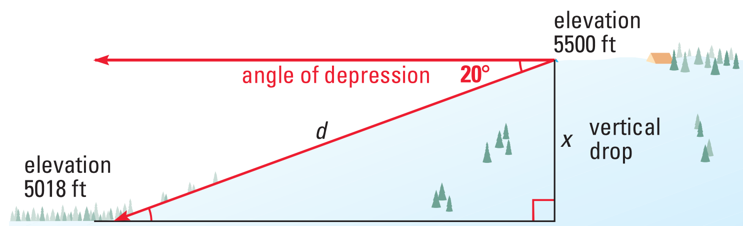
37. **WATER SLIDE** The angle of elevation from the base to the top of a waterslide is about 13°. The slide extends horizontally about 58.2 meters. Estimate the height h of the slide.



38. **SURVEYING** To find the distance d from a house on shore to a house on an island, a surveyor measures from the house on shore to point B, as shown in the diagram. An instrument called a *transit* is used to find the measure of $\angle B$. Estimate the distance d.



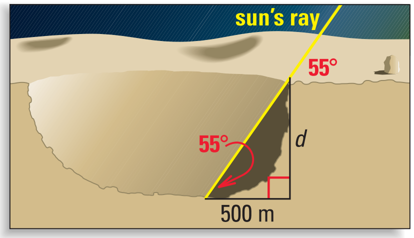
39. **SKI SLOPE** Suppose you stand at the top of a ski slope and look down at the bottom. The angle that your line of sight makes with a line drawn horizontally is called the *angle of depression*, as shown below. The *vertical drop* is the difference in the elevations of the top and the bottom of the slope. Find the vertical drop x of the slope in the diagram. Then estimate the distance d a person skiing would travel on this slope.



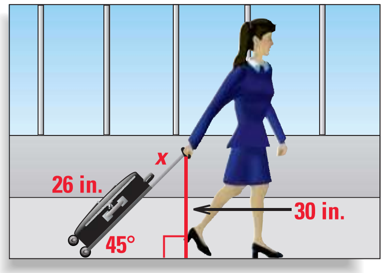
FOCUS ON APPLICATIONS

LUNAR CRATERS Because the moon has no atmosphere to protect it from being hit by meteorites, its surface is pitted with craters. There is no wind, so a crater can remain undisturbed for millions of years—unless another meteorite crashes into it.

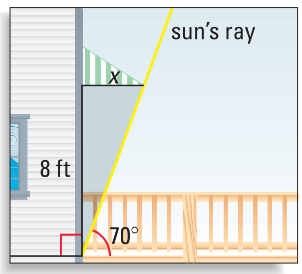
40. **SCIENCE CONNECTION** Scientists can measure the depths of craters on the moon by looking at photos of shadows. The length of the shadow cast by the edge of a crater is about 500 meters. The sun's angle of elevation is 55°. Estimate the depth d of the crater.



41. **LUGGAGE DESIGN** Some luggage pieces have wheels and a handle so that the luggage can be pulled along the ground. Suppose a person's hand is about 30 inches from the floor. About how long should the handle be on the suitcase shown so that it can roll at a comfortable angle of 45° with the floor?

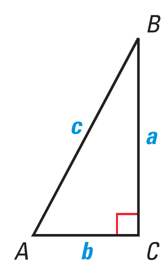


42. **BUYING AN AWNING** Your family room has a sliding-glass door with a southern exposure. You want to buy an awning for the door that will be just long enough to keep the sun out when it is at its highest point in the sky. The angle of elevation of the sun at this point is 70°, and the height of the door is 8 feet. About how far should the overhang extend?



CRITICAL THINKING **In Exercises 43 and 44, use the diagram.**

43. Write expressions for the sine, the cosine, and the tangent of each acute angle in the triangle.

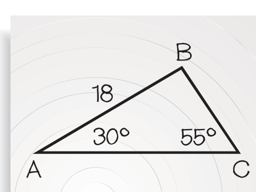


44. *Writing* Use your results from Exercise 43 to explain how the tangent of one acute angle of a right triangle is related to the tangent of the other acute angle. How are the sine and the cosine of one acute angle of a right triangle related to the sine and the cosine of the other acute angle?

45. **TECHNOLOGY** Use geometry software to construct a right triangle. Use your triangle to explore and answer the questions below. Explain your procedure.

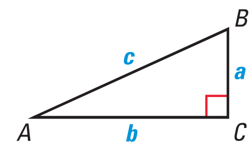
- For what angle measure is the tangent of an acute angle equal to 1?
- For what angle measures is the tangent of an acute angle greater than 1?
- For what angle measures is the tangent of an acute angle less than 1?

46. **ERROR ANALYSIS** To find the length of $\overline{BC}$ in the diagram at the right, a student writes $\tan 55° = \frac{18}{BC}$. What mistake is the student making? Show how the student can find BC. (*Hint:* Begin by drawing an altitude from B to $\overline{AC}$.)



47. **PROOF** Use the diagram of $\triangle ABC$. Complete the proof of the trigonometric identity below.

$$(\sin A)^2 + (\cos A)^2 = 1$$



GIVEN ▶ $\sin A = \frac{a}{c}, \cos A = \frac{b}{c}$

PROVE ▶ $(\sin A)^2 + (\cos A)^2 = 1$

Statements	Reasons
1. $\sin A = \frac{a}{c}, \cos A = \frac{b}{c}$	**1.** ?
2. $a^2 + b^2 = c^2$	**2.** ?
3. $\frac{a^2}{c^2} + \frac{b^2}{c^2} = 1$	**3.** ?
4. $\left(\frac{a}{c}\right)^2 + \left(\frac{b}{c}\right)^2 = 1$	**4.** A property of exponents
5. $(\sin A)^2 + (\cos A)^2 = 1$	**5.** ?

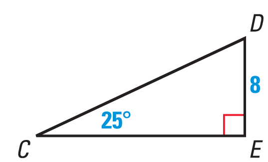
DEMONSTRATING A FORMULA **Show that $(\sin A)^2 + (\cos A)^2 = 1$ for the given angle measure.**

48. $m\angle A = 30°$ **49.** $m\angle A = 45°$ **50.** $m\angle A = 60°$ **51.** $m\angle A = 13°$

52. **PROOF** Use the diagram in Exercise 47. Write a two-column proof of the following trigonometric identity: $\tan A = \frac{\sin A}{\cos A}$.

Test Preparation

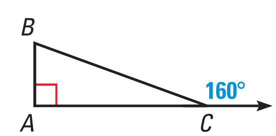
53. **MULTIPLE CHOICE** Use the diagram at the right. Find CD.



Ⓐ $8 \cos 25°$ Ⓑ $8 \sin 25°$ Ⓒ $8 \tan 25°$

Ⓓ $\frac{8}{\sin 25°}$ Ⓔ $\frac{8}{\cos 25°}$

54. **MULTIPLE CHOICE** Use the diagram at the right. Which expression is *not* equivalent to AC?



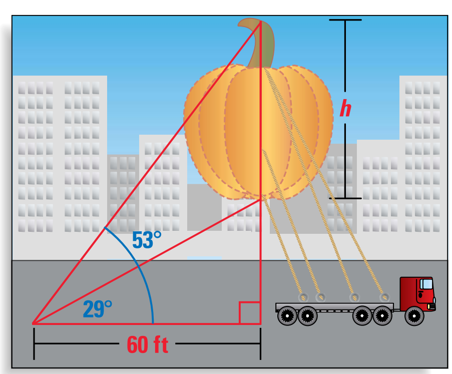
Ⓐ $BC \sin 70°$ Ⓑ $BC \cos 20°$ Ⓒ $\frac{BC}{\tan 20°}$

Ⓓ $\frac{BA}{\tan 20°}$ Ⓔ $BA \tan 70°$

★ Challenge

55. **PARADE** You are at a parade looking up at a large balloon floating directly above the street. You are 60 feet from a point on the street directly beneath the balloon. To see the top of the balloon, you look up at an angle of 53°. To see the bottom of the balloon, you look up at an angle of 29°.

Estimate the height h of the balloon to the nearest foot.

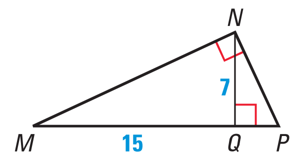


EXTRA CHALLENGE
www.mcdougallittell.com

MIXED REVIEW

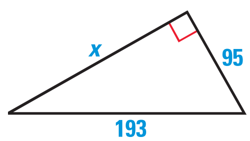
56. SKETCHING A DILATION $\triangle PQR$ is mapped onto $\triangle P'Q'R'$ by a dilation. In $\triangle PQR$, $PQ = 3$, $QR = 5$, and $PR = 4$. In $\triangle P'Q'R'$, $P'Q' = 6$. Sketch the dilation, identify it as a reduction or an enlargement, and find the scale factor. Then find the length of $Q'R'$ and $P'R'$. **(Review 8.7)**

57. FINDING LENGTHS Write similarity statements for the three similar triangles in the diagram. Then find QP and NP. Round decimals to the nearest tenth. **(Review 9.1)**

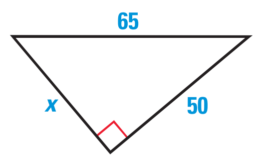


PYTHAGOREAN THEOREM Find the unknown side length. Simplify answers that are radicals. Tell whether the side lengths form a Pythagorean triple. **(Review 9.2 for 9.6)**

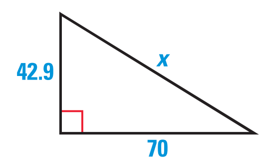
58.



59.



60.



QUIZ 2

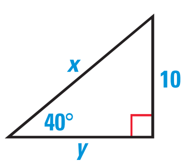
Self-Test for Lessons 9.4 and 9.5

Sketch the figure that is described. Then find the requested information. Round decimals to the nearest tenth. **(Lesson 9.4)**

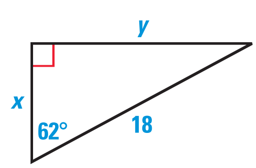
1. The side length of an equilateral triangle is 4 meters. Find the length of an altitude of the triangle.
2. The perimeter of a square is 16 inches. Find the length of a diagonal.
3. The side length of an equilateral triangle is 3 inches. Find the area of the triangle.

Find the value of each variable. Round decimals to the nearest tenth. **(Lesson 9.5)**

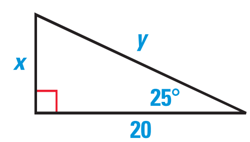
4.



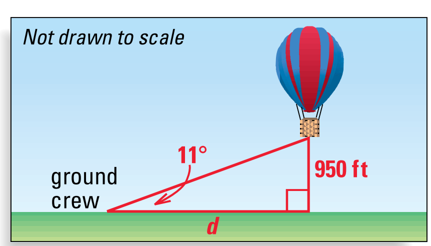
5.



6.



7. HOT-AIR BALLOON The ground crew for a hot-air balloon can see the balloon in the sky at an angle of elevation of 11°. The pilot radios to the crew that the hot-air balloon is 950 feet above the ground. Estimate the horizontal distance d of the hot-air balloon from the ground crew. **(Lesson 9.5)**



9.6

Solving Right Triangles

What *you should learn*

GOAL 1 Solve a right triangle.

GOAL 2 Use right triangles to solve **real-life** problems, such as finding the glide angle and altitude of a space shuttle in **Example 3**.

Why *you should learn it*

▼ To solve **real-life** problems such as determining the correct dimensions of a wheel-chair ramp in **Exs. 39–41**.

REAL LIFE

GOAL 1 SOLVING A RIGHT TRIANGLE

Every right triangle has one right angle, two acute angles, one hypotenuse, and two legs. To **solve a right triangle** means to determine the measures of all six parts. You can solve a right triangle if you know either of the following:

- Two side lengths
- One side length and one acute angle measure

As you learned in Lesson 9.5, you can use the side lengths of a right triangle to find trigonometric ratios for the acute angles of the triangle. As you will see in this lesson, once you know the sine, the cosine, or the tangent of an acute angle, you can use a calculator to find the measure of the angle.

In general, for an acute angle A:

if $\sin A = x$, then $\sin^{-1} x = m\angle A$. ← **The expression $\sin^{-1} x$ is read as "the inverse sine of x."**

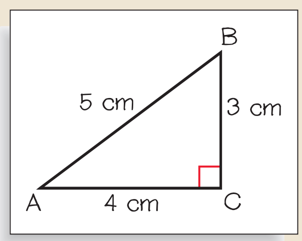
if $\cos A = y$, then $\cos^{-1} y = m\angle A$.

if $\tan A = z$, then $\tan^{-1} z = m\angle A$.

ACTIVITY Developing Concepts

Finding Angles in Right Triangles

1. Carefully draw right $\triangle ABC$ with side lengths of 3 centimeters, 4 centimeters, and 5 centimeters, as shown.



2. Use trigonometric ratios to find the sine, the cosine, and the tangent of $\angle A$. Express the ratios in decimal form.

3. In Step 2, you found that $\sin A = \frac{3}{5} = 0.6$. You can use a calculator to find $\sin^{-1} 0.6$. Most calculators use one of the keystroke sequences below.

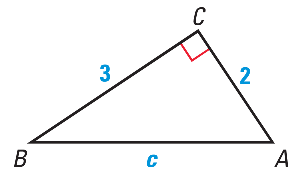
   $\overbrace{\text{2nd}\ \text{SIN}}^{\sin^{-1}}$ 0.6 ENTER or 0.6 $\overbrace{\text{2nd}\ \text{SIN}}^{\sin^{-1}}$

   Make sure your calculator is in degree mode. Then use each of the trigonometric ratios you found in Step 2 to approximate the measure of $\angle A$ to the nearest tenth of a degree.

4. Use a protractor to measure $\angle A$. How does the measured value compare with your calculated values?

EXAMPLE 1 *Solving a Right Triangle*

Solve the right triangle. Round decimals to the nearest tenth.



SOLUTION

Begin by using the Pythagorean Theorem to find the length of the hypotenuse.

$$(\text{hypotenuse})^2 = (\text{leg})^2 + (\text{leg})^2 \qquad \textbf{Pythagorean Theorem}$$

$$c^2 = 3^2 + 2^2 \qquad \textbf{Substitute.}$$

$$c^2 = 13 \qquad \textbf{Simplify.}$$

$$c = \sqrt{13} \qquad \textbf{Find the positive square root.}$$

$$c \approx 3.6 \qquad \textbf{Use a calculator to approximate.}$$

Then use a calculator to find the measure of $\angle B$:

(2 ÷ 3) 2nd TAN $\approx 33.7°$

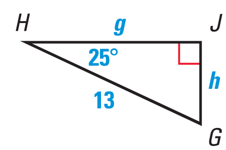
Finally, because $\angle A$ and $\angle B$ are complements, you can write

$$m\angle A = 90° - m\angle B \approx 90° - 33.7° = 56.3°.$$

▸ The side lengths of the triangle are 2, 3, and $\sqrt{13}$, or about 3.6. The triangle has one right angle and two acute angles whose measures are about 33.7° and 56.3°.

EXAMPLE 2 *Solving a Right Triangle*

Solve the right triangle. Round decimals to the nearest tenth.



SOLUTION

Use trigonometric ratios to find the values of g and h.

$$\sin H = \frac{\text{opp.}}{\text{hyp.}} \qquad \cos H = \frac{\text{adj.}}{\text{hyp.}}$$

$$\sin 25° = \frac{h}{13} \qquad \cos 25° = \frac{g}{13}$$

$$13 \sin 25° = h \qquad 13 \cos 25° = g$$

$$13(0.4226) \approx h \qquad 13(0.9063) \approx g$$

$$5.5 \approx h \qquad 11.8 \approx g$$

Because $\angle H$ and $\angle G$ are complements, you can write

$$m\angle G = 90° - m\angle H = 90° - 25° = 65°.$$

▸ The side lengths of the triangle are about 5.5, 11.8, and 13. The triangle has one right angle and two acute angles whose measures are 65° and 25°.

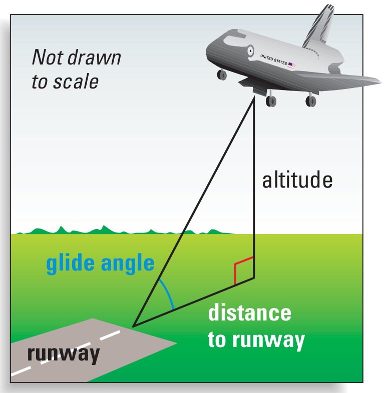
STUDENT HELP

Study Tip
There are other ways to find the side lengths in Examples 1 and 2. For instance, in Example 2, you can use a trigonometric ratio to find one side length, and then use the Pythagorean Theorem to find the other side length.

GOAL 2 USING RIGHT TRIANGLES IN REAL LIFE

EXAMPLE 3 *Solving a Right Triangle*

SPACE SHUTTLE During its approach to Earth, the space shuttle's glide angle changes.



a. When the shuttle's altitude is about 15.7 miles, its horizontal distance to the runway is about 59 miles. What is its glide angle? Round your answer to the nearest tenth.

b. When the space shuttle is 5 miles from the runway, its glide angle is about 19°. Find the shuttle's altitude at this point in its descent. Round your answer to the nearest tenth.

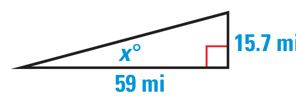
FOCUS ON CAREERS

ASTRONAUT
Some astronauts are pilots who are qualified to fly the space shuttle. Some shuttle astronauts are mission specialists whose responsibilities include conducting scientific experiments in space. All astronauts need to have a strong background in science and mathematics.

CAREER LINK
www.mcdougallittell.com

SOLUTION

a. Sketch a right triangle to model the situation. Let $x°$ = the measure of the shuttle's glide angle. You can use the tangent ratio and a calculator to find the approximate value of x.



$$\tan x° = \frac{\text{opp.}}{\text{adj.}}$$

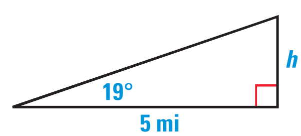
$$\tan x° = \frac{15.7}{59}$$ **Substitute.**

$x =$ (15.7 ÷ 59) 2nd TAN **Use a calculator to find $\tan^{-1}\left(\frac{15.7}{59}\right)$.**

$$x \approx 14.9$$

▶ When the space shuttle's altitude is about 15.7 miles, the glide angle is about 14.9°.

b. Sketch a right triangle to model the situation. Let h = the altitude of the shuttle. You can use the tangent ratio and a calculator to find the approximate value of h.



$$\tan 19° = \frac{\text{opp.}}{\text{adj.}}$$

$$\tan 19° = \frac{h}{5}$$ **Substitute.**

$$0.3443 \approx \frac{h}{5}$$ **Use a calculator.**

$$1.7 \approx h$$ **Multiply each side by 5.**

▶ The shuttle's altitude is about 1.7 miles.

STUDENT HELP

HOMEWORK HELP
Visit our Web site www.mcdougallittell.com for extra examples.

GUIDED PRACTICE

Vocabulary Check ✓

1. Explain what is meant by *solving* a right triangle.

Concept Check ✓

Tell whether the statement is *true* or *false*.

2. You can solve a right triangle if you are given the lengths of any two sides.

3. You can solve a right triangle if you know only the measure of one acute angle.

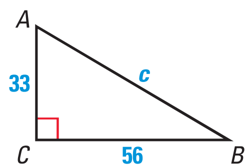
Skill Check ✓

CALCULATOR In Exercises 4–7, $\angle A$ is an acute angle. Use a calculator to approximate the measure of $\angle A$ to the nearest tenth of a degree.

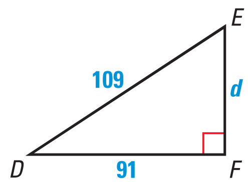
4. $\tan A = 0.7$ **5.** $\tan A = 5.4$ **6.** $\sin A = 0.9$ **7.** $\cos A = 0.1$

Solve the right triangle. Round decimals to the nearest tenth.

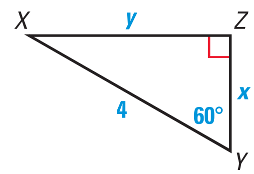
8.



9.



10.



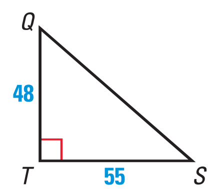
PRACTICE AND APPLICATIONS

STUDENT HELP

Extra Practice
to help you master skills is on p. 820.

FINDING MEASUREMENTS **Use the diagram to find the indicated measurement. Round your answer to the nearest tenth.**

11. QS **12.** $m\angle Q$ **13.** $m\angle S$



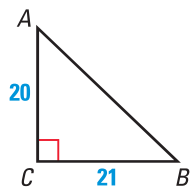
CALCULATOR In Exercises 14–21, $\angle A$ is an acute angle. Use a calculator to approximate the measure of $\angle A$ to the nearest tenth of a degree.

14. $\tan A = 0.5$ **15.** $\tan A = 1.0$ **16.** $\sin A = 0.5$ **17.** $\sin A = 0.35$

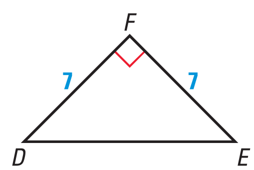
18. $\cos A = 0.15$ **19.** $\cos A = 0.64$ **20.** $\tan A = 2.2$ **21.** $\sin A = 0.11$

SOLVING RIGHT TRIANGLES **Solve the right triangle. Round decimals to the nearest tenth.**

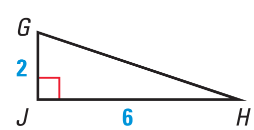
22.



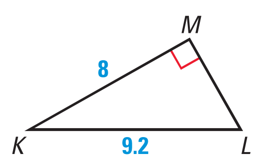
23.



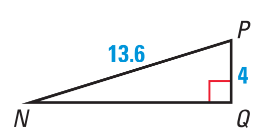
24.



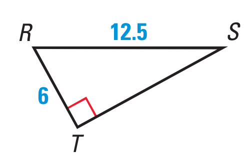
25.



26.



27.

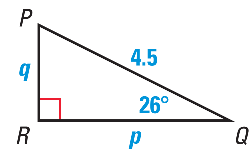


STUDENT HELP

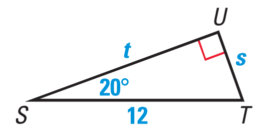
HOMEWORK HELP
Example 1: Exs. 11–27, 34–37
Example 2: Exs. 28–33
Example 3: Exs. 38–41

SOLVING RIGHT TRIANGLES **Solve the right triangle. Round decimals to the nearest tenth.**

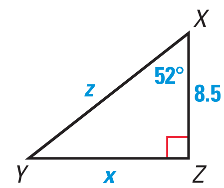
28.



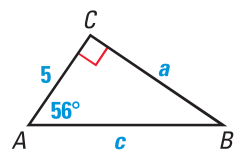
29.



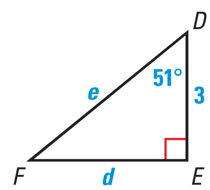
30.



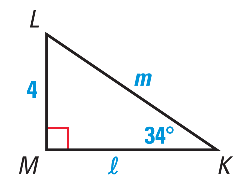
31.



32.



33.



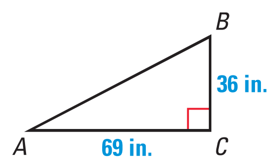
NATIONAL AQUARIUM **Use the diagram of one of the triangular windowpanes at the National Aquarium in Baltimore, Maryland, to find the indicated value.**

34. $\tan B \approx$ __?__

35. $m\angle B \approx$ __?__

36. $AB \approx$ __?__

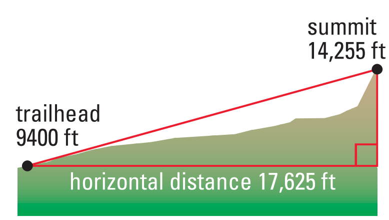
37. $\sin A \approx$ __?__



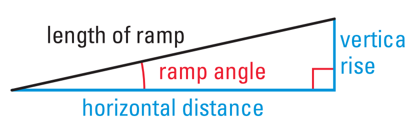
FOCUS ON APPLICATIONS

BENCHMARKS If you hike to the top of a mountain you may find a brass plate called a *benchmark*. A benchmark gives an official elevation for the point that can be used by surveyors as a reference for surveying elevations of other landmarks.

38. **HIKING** You are hiking up a mountain peak. You begin hiking at a trailhead whose elevation is about 9400 feet. The trail ends near the summit at 14,255 feet. The horizontal distance between these two points is about 17,625 feet. Estimate the angle of elevation from the trailhead to the summit.



RAMPS **In Exercises 39–41, use the information about wheelchair ramps.** The Uniform Federal Accessibility Standards specify that the ramp angle used for a wheelchair ramp must be less than or equal to 4.76°.



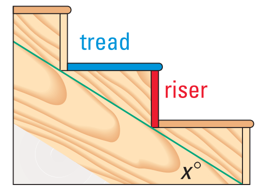
39. The length of one ramp is 20 feet. The vertical rise is 17 inches. Estimate the ramp's horizontal distance and its ramp angle.

40. You want to build a ramp with a vertical rise of 8 inches. You want to minimize the horizontal distance taken up by the ramp. Draw a sketch showing the approximate dimensions of your ramp.

41. *Writing* Measure the horizontal distance and the vertical rise of a ramp near your home or school. Find the ramp angle. Does the ramp meet the specifications described above? Explain.

Test Preparation

MULTI-STEP PROBLEM In Exercises 42–45, use the diagram and the information below. The horizontal part of a step is called the *tread*. The vertical part is called the *riser*. The ratio of the riser length to the tread length affects the safety of a staircase. Traditionally, builders have used a riser-to-tread ratio of about $8\frac{1}{4}$ inches : 9 inches. A newly recommended ratio is 7 inches : 11 inches.



42. Find the value of x for stairs built using the new riser-to-tread ratio.

43. Find the value of x for stairs built using the old riser-to-tread ratio.

44. Suppose you want to build a stairway that is less steep than either of the ones in Exercises 42 and 43. Give an example of a riser-to-tread ratio that you could use. Find the value of x for your stairway.

45. *Writing* Explain how the riser-to-tread ratio that is used for a stairway could affect the safety of the stairway.

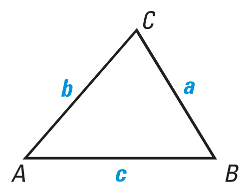
★ Challenge

46. PROOF Write a proof.

GIVEN ▸ $\angle A$ and $\angle B$ are acute angles.

PROVE ▸ $\dfrac{a}{\sin A} = \dfrac{b}{\sin B}$

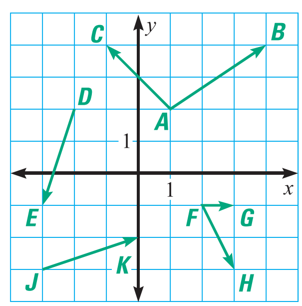
(*Hint:* Draw an altitude from C to $\overline{AB}$. Label it h.)



EXTRA CHALLENGE
www.mcdougallittell.com

MIXED REVIEW

USING VECTORS Write the component form of the vector. (Review 7.4 for 9.7)



47. $\overrightarrow{AB}$ **48.** $\overrightarrow{AC}$

49. $\overrightarrow{DE}$ **50.** $\overrightarrow{FG}$

51. $\overrightarrow{FH}$ **52.** $\overrightarrow{JK}$

SOLVING PROPORTIONS Solve the proportion. (Review 8.1)

53. $\frac{x}{30} = \frac{5}{6}$ **54.** $\frac{7}{16} = \frac{49}{y}$ **55.** $\frac{3}{10} = \frac{g}{42}$

56. $\frac{7}{18} = \frac{84}{k}$ **57.** $\frac{m}{2} = \frac{7}{1}$ **58.** $\frac{8}{t} = \frac{4}{11}$

CLASSIFYING TRIANGLES Decide whether the numbers can represent the side lengths of a triangle. If they can, classify the triangle as *right, acute,* or *obtuse.* (Review 9.3)

59. 18, 14, 2 **60.** 60, 228, 220 **61.** 8.5, 7.7, 3.6

62. 250, 263, 80 **63.** 113, 15, 112 **64.** 15, 75, 59

9.7 Vectors

What *you should learn*

GOAL 1 Find the magnitude and the direction of a vector.

GOAL 2 Add vectors.

Why *you should learn it*

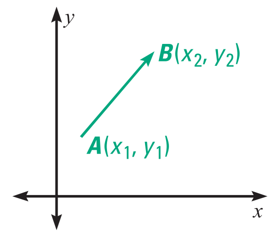
▼ To solve **real-life** problems, such as describing the velocity of a skydiver in **Exs. 41–45**.

GOAL 1 FINDING THE MAGNITUDE OF A VECTOR

As defined in Lesson 7.4, a *vector* is a quantity that has both magnitude and direction. In this lesson, you will learn how to find the *magnitude of a vector* and the *direction of a vector*. You will also learn how to add vectors.

The **magnitude of a vector** $\overrightarrow{AB}$ is the distance from the initial point A to the terminal point B, and is written $|\overrightarrow{AB}|$. If a vector is drawn in a coordinate plane, you can use the Distance Formula to find its magnitude.

$$|\overrightarrow{AB}| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$



EXAMPLE 1 *Finding the Magnitude of a Vector*

Points P and Q are the initial and terminal points of the vector $\overrightarrow{PQ}$. Draw $\overrightarrow{PQ}$ in a coordinate plane. Write the component form of the vector and find its magnitude.

a. $P(0, 0), Q(-6, 3)$ **b.** $P(0, 2), Q(5, 4)$ **c.** $P(3, 4), Q(-2, -1)$

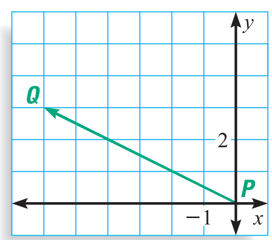
SOLUTION

a. Component form $= \langle x_2 - x_1, y_2 - y_1 \rangle$

$$\overrightarrow{PQ} = \langle -6 - 0, 3 - 0 \rangle = \langle -6, 3 \rangle$$

Use the Distance Formula to find the magnitude.

$$|\overrightarrow{PQ}| = \sqrt{(-6 - 0)^2 + (3 - 0)^2} = \sqrt{45} \approx 6.7$$



STUDENT HELP

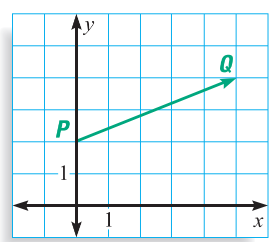
Look Back
For help with the component form of a vector, see p. 423.

b. Component form $= \langle x_2 - x_1, y_2 - y_1 \rangle$

$$\overrightarrow{PQ} = \langle 5 - 0, 4 - 2 \rangle = \langle 5, 2 \rangle$$

Use the Distance Formula to find the magnitude.

$$|\overrightarrow{PQ}| = \sqrt{(5 - 0)^2 + (4 - 2)^2} = \sqrt{29} \approx 5.4$$

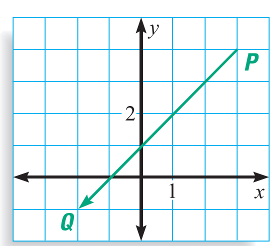


c. Component form $= \langle x_2 - x_1, y_2 - y_1 \rangle$

$$\overrightarrow{PQ} = \langle -2 - 3, -1 - 4 \rangle = \langle -5, -5 \rangle$$

Use the Distance Formula to find the magnitude.

$$|\overrightarrow{PQ}| = \sqrt{(-2 - 3)^2 + (-1 - 4)^2} = \sqrt{50} \approx 7.1$$



FOCUS ON APPLICATIONS

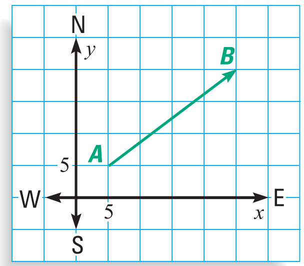
REAL LIFE

NAVIGATION One of the most common vector quantities in real life is the velocity of a moving object. A velocity vector is used in navigation to describe both the speed and the direction of a moving object.

The **direction of a vector** is determined by the angle it makes with a horizontal line. In real-life applications, the direction angle is described relative to the directions north, east, south, and west. In a coordinate plane, the x-axis represents an east-west line. The y-axis represents a north-south line.

EXAMPLE 2 *Describing the Direction of a Vector*

The vector $\overrightarrow{AB}$ describes the velocity of a moving ship. The scale on each axis is in miles per hour.



a. Find the speed of the ship.

b. Find the direction it is traveling relative to east.

SOLUTION

a. The magnitude of the vector $\overrightarrow{AB}$ represents the ship's speed. Use the Distance Formula.

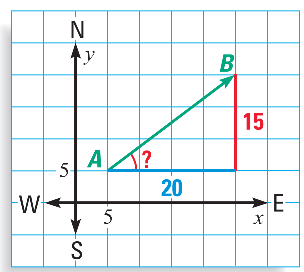
$$\left|\overrightarrow{AB}\right| = \sqrt{(25-5)^2 + (20-5)^2}$$
$$= \sqrt{20^2 + 15^2}$$
$$= 25$$

▶ The speed of the ship is 25 miles per hour.

STUDENT HELP

Look Back For help with using trigonometric ratios to find angle measures, see pp. 567 and 568.

b. The tangent of the angle formed by the vector and a line drawn parallel to the x-axis is $\frac{15}{20}$, or 0.75. Use a calculator to find the angle measure.



0.75 [2nd] [TAN] $\approx 36.9°$

▶ The ship is traveling in a direction about 37° north of east.

. . . .

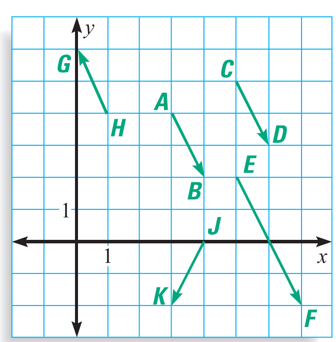
Two vectors are **equal** if they have the same magnitude and direction. They do *not* have to have the same initial and terminal points. Two vectors are **parallel** if they have the same or opposite directions.

EXAMPLE 3 *Identifying Equal and Parallel Vectors*

In the diagram, these vectors have the same direction: $\overrightarrow{AB}$, $\overrightarrow{CD}$, $\overrightarrow{EF}$.

These vectors are equal: $\overrightarrow{AB}$, $\overrightarrow{CD}$.

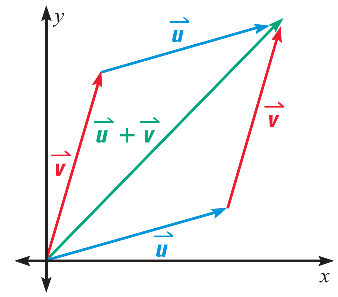
These vectors are parallel: $\overrightarrow{AB}$, $\overrightarrow{CD}$, $\overrightarrow{EF}$, $\overrightarrow{HG}$.



GOAL 2 ADDING VECTORS

> **STUDENT HELP**
>
> **Study Tip**
> A single letter with an arrow over it, such as $\vec{u}$, can be used to denote a vector.

Two vectors can be added to form a new vector. To add $\vec{u}$ and $\vec{v}$ geometrically, place the initial point of $\vec{v}$ on the terminal point of $\vec{u}$, (or place the initial point of $\vec{u}$ on the terminal point of $\vec{v}$). The sum is the vector that joins the initial point of the first vector and the terminal point of the second vector .



This method of adding vectors is often called the *parallelogram rule* because the sum vector is the diagonal of a parallelogram. You can also add vectors algebraically.

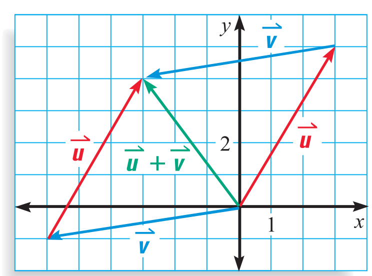
ADDING VECTORS

SUM OF TWO VECTORS

The **sum** of $\vec{u} = \langle a_1, b_1 \rangle$ and $\vec{v} = \langle a_2, b_2 \rangle$ is $\vec{u} + \vec{v} = \langle a_1 + a_2, b_1 + b_2 \rangle$.

EXAMPLE 4 *Finding the Sum of Two Vectors*

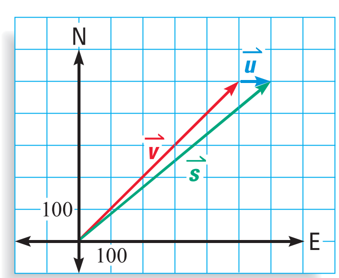
Let $\vec{u} = \langle 3, 5 \rangle$ and $\vec{v} = \langle -6, -1 \rangle$. To find the sum vector $\vec{u} + \vec{v}$, add the horizontal components and add the vertical components of $\vec{u}$ and $\vec{v}$.



$$\vec{u} + \vec{v} = \langle 3 + (-6), 5 + (-1) \rangle$$
$$= \langle -3, 4 \rangle$$

EXAMPLE 5 *Velocity of a Jet*

AVIATION A jet is flying northeast at about 707 miles per hour. Its velocity is represented by the vector $\vec{v} = \langle 500, 500 \rangle$.



The jet encounters a wind blowing from the west at 100 miles per hour. The wind velocity is represented by $\vec{u} = \langle 100, 0 \rangle$. The jet's new velocity vector $\vec{s}$ is the sum of its original velocity vector and the wind's velocity vector.

$$\vec{s} = \vec{v} + \vec{u}$$
$$= \langle 500 + 100, 500 + 0 \rangle$$
$$= \langle 600, 500 \rangle$$

The magnitude of the sum vector $\vec{s}$ represents the new speed of the jet.

$$\text{New speed} = |\vec{s}| = \sqrt{(600 - 0)^2 + (500 - 0)^2} \approx 781 \text{ mi/h}$$

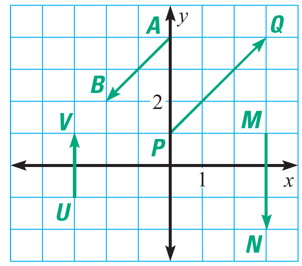
GUIDED PRACTICE

Vocabulary Check ✓

1. What is meant by the *magnitude of a vector* and the *direction of a vector*?

Concept Check ✓

In Exercises 2–4, use the diagram.



2. Write the component form of each vector.

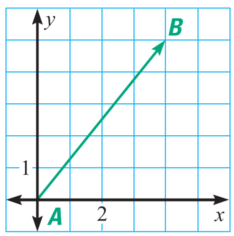
3. Identify any parallel vectors.

4. Vectors $\overrightarrow{PQ}$ and $\overrightarrow{ST}$ are equal vectors. Although $\overrightarrow{ST}$ is not shown, the coordinates of its initial point are $(-1, -1)$. Give the coordinates of its terminal point.

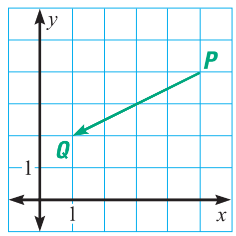
Skill Check ✓

Write the vector in component form. Find the magnitude of the vector. Round your answer to the nearest tenth.

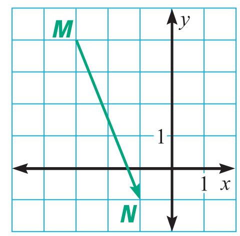
5.



6.



7.



8. Use the vector in Exercise 5. Find the direction of the vector relative to east.

9. Find the sum of the vectors in Exercises 5 and 6.

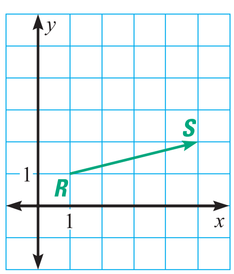
PRACTICE AND APPLICATIONS

STUDENT HELP

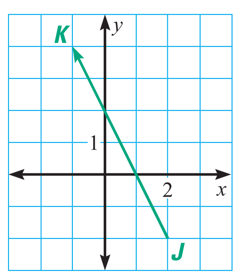
Extra Practice
to help you master skills is on p. 820.

FINDING MAGNITUDE **Write the vector in component form. Find the magnitude of the vector. Round your answer to the nearest tenth.**

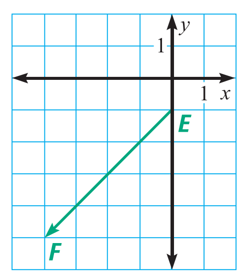
10.



11.



12.



FINDING MAGNITUDE **Draw vector $\overrightarrow{PQ}$ in a coordinate plane. Write the component form of the vector and find its magnitude. Round your answer to the nearest tenth.**

13. $P(0, 0), Q(2, 7)$

14. $P(5, 1), Q(2, 6)$

15. $P(-3, 2), Q(7, 6)$

16. $P(-4, -3), Q(2, -7)$

17. $P(5, 0), Q(-1, -4)$

18. $P(6, 3), Q(-2, 1)$

19. $P(-6, 0), Q(-5, -4)$

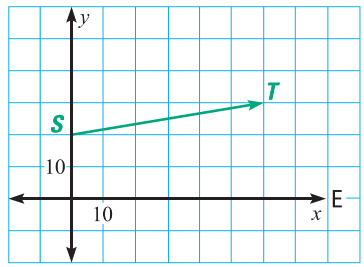
20. $P(0, 5), Q(3, 5)$

STUDENT HELP

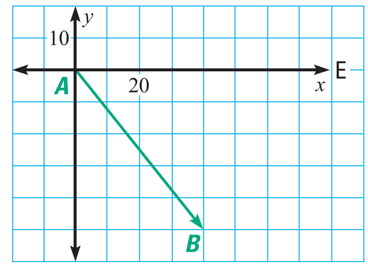
HOMEWORK HELP
Example 1: Exs. 10–20
Example 2: Exs. 21–24
Example 3: Exs. 25–29
Example 4: Exs. 31–40
Example 5: Exs. 41–45

NAVIGATION **The given vector represents the velocity of a ship at sea. Find the ship's speed, rounded to the nearest mile per hour. Then find the direction the ship is traveling relative to the given direction.**

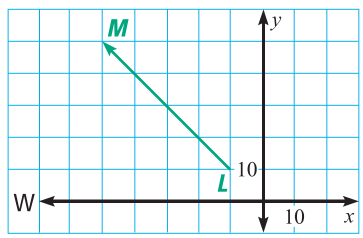
21. Find direction relative to east.



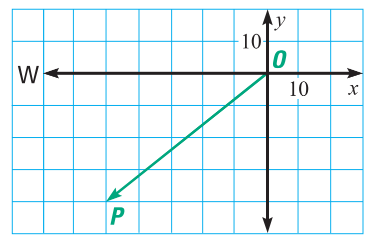
22. Find direction relative to east.



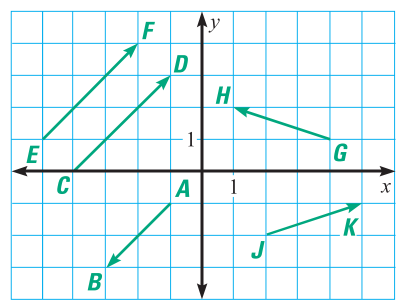
23. Find direction relative to west.



24. Find direction relative to west.



PARALLEL AND EQUAL VECTORS
In Exercises 25–28, use the diagram shown at the right.



25. Which vectors are parallel?

26. Which vectors have the same direction?

27. Which vectors are equal?

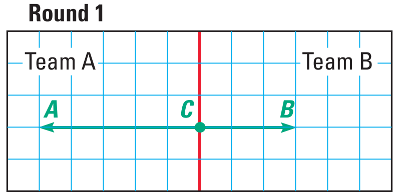
28. Name two vectors that have the same magnitude but different directions.

FOCUS ON APPLICATIONS

REAL LIFE **TUG-OF-WAR** In the game *tug-of-war*, two teams pull on opposite ends of a rope. The team that succeeds in pulling a member of the other team across a center line wins.

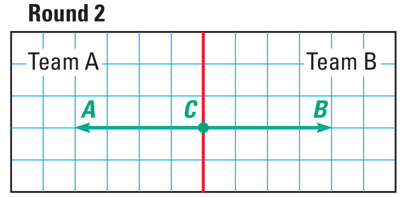
TUG-OF-WAR GAME **In Exercises 29 and 30, use the information below.** The forces applied in a game of *tug-of-war* can be represented by vectors. The magnitude of the vector represents the amount of force with which the rope is pulled. The direction of the vector represents the direction of the pull. The diagrams below show the forces applied in two different rounds of tug-of-war.

Round 1



center line

Round 2



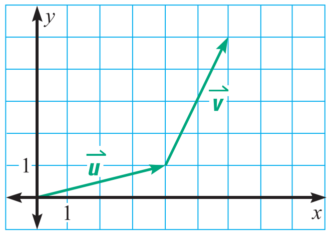
center line

29. In Round 2, are $\overrightarrow{CA}$ and $\overrightarrow{CB}$ parallel vectors? Are they equal vectors?

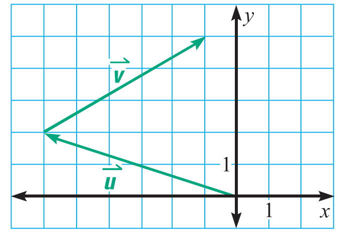
30. In which round was the outcome a tie? How do you know? Describe the outcome in the other round. Explain your reasoning.

PARALLELOGRAM RULE **Copy the vectors $\vec{u}$ and $\vec{v}$. Write the component form of each vector. Then find the sum $\vec{u} + \vec{v}$ and draw the vector $\vec{u} + \vec{v}$.**

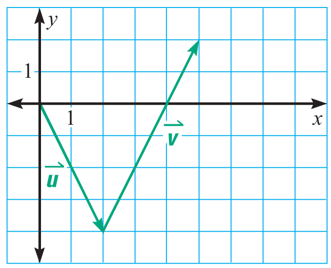
31.



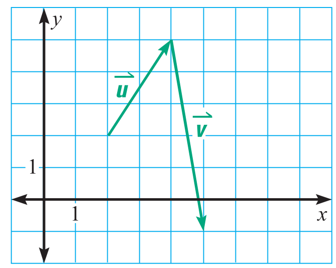
32.



33.



34.



ADDING VECTORS **Let $\vec{u} = \langle 7, 3\rangle$, $\vec{v} = \langle 1, 4\rangle$, $\vec{w} = \langle 3, 7\rangle$, and $\vec{z} = \langle -3, -7\rangle$. Find the given sum.**

35. $\vec{v} + \vec{w}$

36. $\vec{u} + \vec{v}$

37. $\vec{u} + \vec{w}$

38. $\vec{v} + \vec{z}$

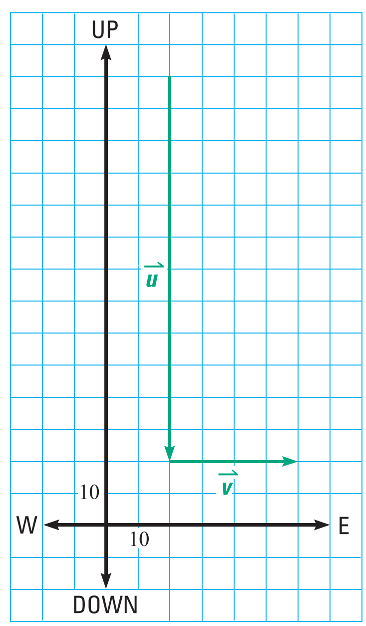
39. $\vec{u} + \vec{z}$

40. $\vec{w} + \vec{z}$

FOCUS ON APPLICATIONS

SKYDIVING A skydiver who has not yet opened his or her parachute is in *free fall*. During free fall, the skydiver accelerates at first. Air resistance eventually stops this acceleration, and the skydiver falls at *terminal velocity*.

SKYDIVING **In Exercises 41–45, use the information and diagram below.** A skydiver is falling at a constant downward velocity of 120 miles per hour. In the diagram, vector $\vec{u}$ represents the skydiver's velocity. A steady breeze pushes the skydiver to the east at 40 miles per hour. Vector $\vec{v}$ represents the wind velocity. The scales on the axes of the graph are in miles per hour.



41. Write the vectors $\vec{u}$ and $\vec{v}$ in component form.

42. Let $\vec{s} = \vec{u} + \vec{v}$. Copy the diagram and draw vector $\vec{s}$.

43. Find the magnitude of $\vec{s}$. What information does the magnitude give you about the skydiver's fall?

44. If there were no wind, the skydiver would fall in a path that was straight down. At what angle to the ground is the path of the skydiver when the skydiver is affected by the 40 mile per hour wind from the west?

45. Suppose the skydiver was blown to the west at 30 miles per hour. Sketch a new diagram and find the skydiver's new velocity.

46. *Writing* Write the component form of a vector with the same magnitude as $\overrightarrow{JK} = \langle 1, 3\rangle$ but a different direction. Explain how you found the vector.

47. **LOGICAL REASONING** Let vector $\vec{u} = \langle r, s\rangle$. Suppose the horizontal and the vertical components of $\vec{u}$ are multiplied by a constant k. The resulting vector is $\vec{v} = \langle kr, ks\rangle$. How are the magnitudes and the directions of $\vec{u}$ and $\vec{v}$ related when k is positive? when k is negative? Justify your answers.

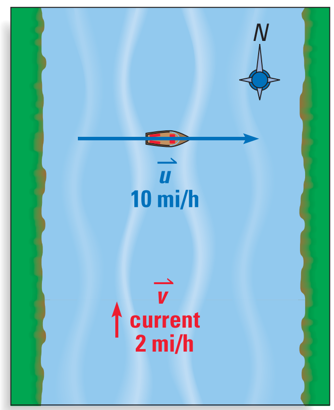
Test Preparation

48. **MULTI-STEP PROBLEM** A motorboat heads due east across a river at a speed of 10 miles per hour. Vector $\vec{u} = \langle 10, 0\rangle$ represents the velocity of the motorboat. The current of the river is flowing due north at a speed of 2 miles per hour. Vector $\vec{v} = \langle 0, 2\rangle$ represents the velocity of the current.

a. Let $\vec{s} = \vec{u} + \vec{v}$. Draw the vectors $\vec{u}$, $\vec{v}$, and $\vec{s}$ in a coordinate plane.

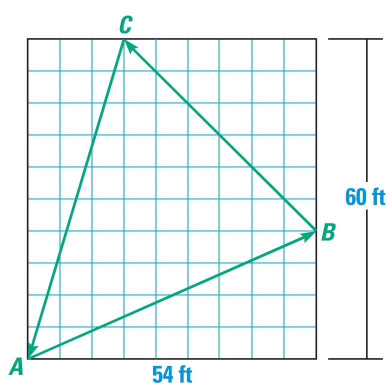
b. Find the speed and the direction of the motorboat as it is affected by the current.

c. Suppose the speed of the motorboat is greater than 10 miles per hour, and the speed of the current is less than 2 miles per hour. Describe one possible set of vectors $\vec{u}$ and $\vec{v}$ that could represent the velocity of the motorboat and the velocity of the current. Write and solve a word problem that can be solved by finding the sum of the two vectors.



Challenge

BUMPER CARS **In Exercises 49–52, use the information below.**
As shown in the diagram below, a bumper car moves from point A to point B to point C and back to point A. The car follows the path shown by the vectors. The magnitude of each vector represents the distance traveled by the car from the initial point to the terminal point.



49. Find the sum of $\overrightarrow{AB}$ and $\overrightarrow{BC}$. Write the sum vector in component form.

50. Add vector $\overrightarrow{CA}$ to the sum vector from Exercise 49.

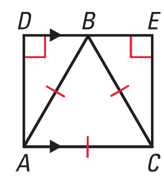
51. Find the total distance traveled by the car.

52. Compare your answers to Exercises 50 and 51. Why are they different?

EXTRA CHALLENGE
www.mcdougallittell.com

MIXED REVIEW

53. **PROOF** Use the information and the diagram to write a proof. **(Review 4.5)**

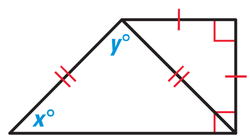


GIVEN ▶ $\angle D$ and $\angle E$ are right angles; $\triangle ABC$ is equilateral; $\overline{DE} \parallel \overline{AC}$

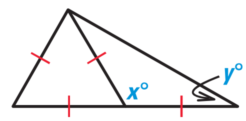
PROVE ▶ B is the midpoint of $\overline{DE}$.

USING ALGEBRA **Find the values of *x* and *y*.** **(Review 4.6)**

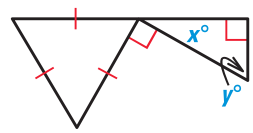
54.



55.



56.



USING ALGEBRA **Find the product.** **(Skills Review, p. 798, for 10.1)**

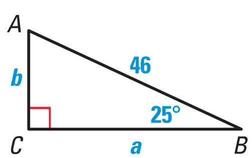
57. $(x + 1)^2$ **58.** $(x + 7)^2$ **59.** $(x + 11)^2$ **60.** $(7 + x)^2$

QUIZ 3

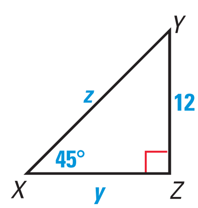
Self-Test for Lessons 9.6 and 9.7

Solve the right triangle. Round decimals to the nearest tenth. **(Lesson 9.6)**

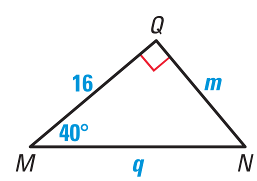
1.



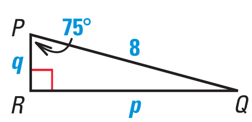
2.



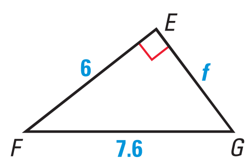
3.



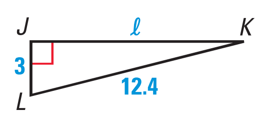
4.



5.



6.



Draw vector $\overrightarrow{PQ}$ in a coordinate plane. Write the component form of the vector and find its magnitude. Round your answer to the nearest tenth. **(Lesson 9.7)**

7. $P(3, 4), Q(-2, 3)$

8. $P(-2, 2), Q(4, -3)$

9. $P(0, -1), Q(3, 4)$

10. $P(2, 6), Q(-5, -5)$

11. Vector $\overrightarrow{ST} = \langle 3, 8 \rangle$. Draw $\overrightarrow{ST}$ in a coordinate plane and find its direction relative to east. **(Lesson 9.7)**

Let $\vec{u} = \langle 0, -5 \rangle$, $\vec{v} = \langle 4, 7 \rangle$, $\vec{w} = \langle -2, -3 \rangle$, and $\vec{z} = \langle 2, 6 \rangle$. Find the given sum. **(Lesson 9.7)**

12. $\vec{u} + \vec{v}$ **13.** $\vec{v} + \vec{w}$ **14.** $\vec{u} + \vec{w}$

15. $\vec{u} + \vec{z}$ **16.** $\vec{v} + \vec{z}$ **17.** $\vec{w} + \vec{z}$